Hadronic Spectra and Kaluza-Klein Picture of the World [1]

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Abstract

A manifestation of Kaluza-Klein picture in hadronic spectra is discussed. We argue that the experimentally observed structures in hadronic spectra confirm the Kaluza-Klein picture of the world.

"... the simpler the presentation of a particular law of Nature, the more general it is ..."

Max Planck, Nobel Lecture, June 2, 1920

1 Introduction

Dear Colleagues.

It seems that here is just the place where we could remember one of the greatest physicists of the XXth century, I mean German physicist Max Planck. My experience in science allows me to definitely share Max Planck's opinion in the above written fragment of his Nobel Lecture. Following this opinion, I'd like to present here a new, very simple and at the same time quite general, physical law concerning the structure of hadron spectra.

Although the modern strong interaction theory formulated in terms of the known QCD Lagrangian is commonly accepted, this theory does not allow to make an appreciable breakthrough in the problem of calculating the masses of compound systems so far, mainly because that problem is significantly non-perturbative. In other words, this means that our theoretical understanding of low-energy QCD spectroscopy is far from what is desired. Even the best currently performed lattice computations in QCD cannot help us to understand the exact nature of the real hadron spectrum.

All of you know that strong interactions are characterized by multi-particle production. The dynamics of multi-particle systems necessarily involve so called many-body forces. Many-body forces are fundamental forces arising in multi-particle systems with more than two particles, and they are responsible for the dynamics of production processes. For

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example, the three-body forces are responsible for the dynamics of one-particle inclusive reactions; see Ref. [1] and references therein. A description of many-body forces requires the use of multidimensional spaces, and, as a consequence, the strong interactions theory cannot be constructed consistently if multidimensional spaces are not used. Therefore, it seems natural to formulate the strong interactions theory in a multidimensional space from the very beginning.

The idea to use multidimensional spaces in fundamental physics is not new: famous works by Kaluza and Klein were the first to introduce this idea. The original idea of Kaluza and Klein is based on the hypothesis that the input space-time is a $(4+d)$-dimensional space $\mathcal{M}_{(4+d)}$ which can be represented as a tensor product of the visible four-dimensional world M_4 with a compact internal d-dimensional space $\mathcal{K}_d$

$$\mathcal{M}_{(4+d)} = M_4 \times \mathcal{K}_d. \tag{1}$$

The compact internal space $\mathcal{K}_d$ is space-like, i.e. it has only spatial dimensions which may be considered as extra spatial dimensions of M_4. An especial example of $\mathcal{M}_{(4+d)}$ is a space with a factorizable metric. According to the tensor product structure of the space $\mathcal{M}_{(4+d)}$, the metric may be chosen in a factorizable form. This means that if $z^M = \{x^\mu, y^m\}$, ($M = 0, 1, \ldots, 3+d$, $\mu = 0, 1, 2, 3$, $m = 1, 2, \ldots, d$) are local coordinates on $\mathcal{M}_{(4+d)}$, then the factorizable metric looks like

$$ds^2 = \mathcal{G}_{MN}(z)dz^M dz^N = g_{\mu\nu}(x)dx^\mu dx^\nu + \gamma_{mn}(x,y)dy^m dy^n,$$

where $g_{\mu\nu}(x)$ is the metric on M_4.

In the year 1921, Kaluza proposed a unification of the theory of gravity and Maxwell theory of electromagnetism in four dimensions starting from the theory of gravity in five dimensions. He assumed that the five-dimensional space $\mathcal{M}_5$ had to be the product of the four-dimensional space-time M_4 and a circle $\mathcal{S}_1$: $\mathcal{M}_5 = M_4 \times \mathcal{S}_1$. It was shown that the zero mode sector of the Kaluza model is equivalent to the four-dimensional theory which describes the Hilbert-Einstein gravity with four-dimensional general coordinate transformations and the Maxwell theory of electromagnetism with gauge transformations.

Recently, some models with extra dimensions have been proposed to attack the electroweak quantum instability of the Standard Model known as the hierarchy problem between electroweak and gravity scales. However, it is obvious that the basic idea of the Kaluza-Klein scenario may be applied to any model in the Quantum Field Theory. As an illustrative example, let us consider the simplest case of the (4+d)-dimensional model of scalar field with the action

$$S = \int d^{4+d}z\sqrt{-\mathcal{G}}\left[\frac{1}{2}\left(\partial_M\Phi\right)^2 - \frac{m^2}{2}\Phi^2 + \frac{G_{(4+d)}}{4!}\Phi^4\right], \tag{2}$$

where $\mathcal{G} = \det|\mathcal{G}_{MN}|$, $\mathcal{G}_{MN}$ is the metric on $\mathcal{M}_{(4+d)} = M_4 \times \mathcal{K}_d$, M_4 is the pseudo-Euclidean Minkowski space-time, $\mathcal{K}_d$ is a compact internal d-dimensional space with the characteristic size R. Let $\Delta_{\mathcal{K}_d}$ be the Laplace operator on the internal space $\mathcal{K}_d$, and $Y_n(y)$ ortho-normalized eigenfunctions of the Laplace operator

$$\Delta_{\mathcal{K}_d} Y_n(y) = -\frac{\lambda_n}{R^2} Y_n(y), \tag{3}$$

here n is the (multi)index labelling the eigenvalue λ_n of the eigenfunction $Y_n(y)$. A d-dimensional torus $\mathcal{T}^d$ with equal radii R is an especially simple example of the compact

internal space of extra dimensions $\mathcal{K}_d$. The eigenfunctions and eigenvalues in this special case look like

$$Y_n(y) = \frac{1}{\sqrt{V_d}} \exp\left(i \sum_{m=1}^{d} n_m y^m / R \right), \tag{4}$$

$$\lambda_n = |n|^2, \quad |n|^2 = n_1^2 + n_2^2 + \ldots n_d^2, \quad n = (n_1, n_2, \ldots, n_d), \quad -\infty \leq n_m \leq \infty,$$

where n_m are integer numbers, $V_d = (2\pi R)^d$ is the volume of the torus.

To reduce the multidimensional theory to the effective four-dimensional one we write a harmonic expansion for the multidimensional field $\Phi(z)$

$$\Phi(z) = \Phi(x, y) = \sum_n \phi^{(n)}(x) Y_n(y). \tag{5}$$

The coefficients $\phi^{(n)}(x)$ of harmonic expansion (5) are called Kaluza-Klein (KK) excitations or KK modes, and they usually include the zero-mode $\phi^{(0)}(x)$, corresponding to $n = 0$ and the eigenvalue $\lambda_0 = 0$. Substitution of the KK mode expansion into action (2) and integration over the internal space $\mathcal{K}_d$ gives

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \left(\partial_\mu \phi^{(0)} \right)^2 - \frac{m^2}{2} (\phi^{(0)})^2 + \frac{g}{4!} (\phi^{(0)})^4 + \right. \tag{6}$$

$$\left. + \sum_{n \neq 0} \left[\frac{1}{2} \left(\partial_\mu \phi^{(n)} \right) \left(\partial^\mu \phi^{(n)} \right)^* - \frac{m_n^2}{2} \phi^{(n)} \phi^{(n)*} \right] + \frac{g}{4!} (\phi^{(0)})^2 \sum_{n \neq 0} \phi^{(n)} \phi^{(n)*} \right\} + \ldots .$$

For the masses of the KK modes one obtains

$$\boxed{m_n^2 = m^2 + \frac{\lambda_n}{R^2}}, \tag{7}$$

and the coupling constant g of the four-dimensional theory is related to the coupling constant $G_{(4+d)}$ of the initial multidimensional theory by the equation

$$\boxed{g = \frac{G_{(4+d)}}{V_d}}, \tag{8}$$

where V_d is the volume of the compact internal space of extra dimensions $\mathcal{K}_d$. The fundamental coupling constant $G_{(4+d)}$ has dimension $[\text{mass}]^{-d}$. So, the four-dimensional coupling constant g is dimensionless, as it should be. Eqs. (7,8) represent the basic relations of the Kaluza-Klein scenario. Similar relations take place for other types of multidimensional quantum field theoretical models. From the four-dimensional point of view we can interpret each KK mode as a particle with the mass m_n given by Eq. (7). Clearly, according to the Kaluza-Klein scenario any multidimensional field contains an infinite set of KK modes, i.e. an infinite set of four-dimensional particles with increasing masses, which is called the Kaluza-Klein tower. Therefore, an experimental observation of the series of KK excitations with a characteristic spectrum of form (7) would be an evidence of the existence of extra dimensions. So far, the KK partners of the particles of the Standard Model have not been observed. In the Kaluza-Klein scenario this fact can be explained by a microscopically small size R of extra dimensions ($R < 10^{-17}\,cm$); in that case the KK excitations may be produced only at super-high energies of the scale

$E \sim 1/R > 1\,TeV$. Below this scale, only homogeneous zero modes with $n = 0$ were accessible for observation in recent high energy experiments. That is why, there is a hope to search the KK excitations at the future LHC and other colliders.

As we have calculated before [2]

$$\frac{1}{R} = 41.481\text{MeV}, \tag{9}$$

or

$$\boxed{R = 24.1\,GeV^{-1} = 4.75\,10^{-13}\text{cm}}. \tag{10}$$

If we relate the strong interaction scale with the pion mass

$$G_{(4+d)} \sim \frac{10}{[m_\pi]^d}, \tag{11}$$

then

$$g \sim \frac{10}{(2\pi m_\pi R)^d}, \tag{12}$$

and

$$g(d=1) \sim 0.5.$$

On the other hand

$$g_{eff} = g_{\pi NN}\exp(-m_\pi R) \sim 0.5, \quad (g^2_{\pi NN}/4\pi = 14.6). \tag{13}$$

So, R has a clear physical meaning: size (10) just corresponds to the scale of distances where strong Yukawa forces in strength come close to electromagnetic ones. Moreover,

$$M \sim R^{-1}\left(M_{Pl}/R^{-1}\right)^{2/(d+2)}|_{d=6} \sim 5\,\text{TeV}. \tag{14}$$

Mass scale (14) is just the scale accepted in the Standard Model, and this is an interesting observation as well. Actually, mass scale (14) means that gravity effects may be detected at the future LHC collider.

2 Peculiarities of Kaluza-Klein excitations

From the formula for the masses of the KK modes

$$m_n = \sqrt{m^2 + \frac{n^2}{R^2}}$$

we obtain

$$m_n = m + \delta m_n, \quad \delta m_n = \frac{n^2}{2mR^2}, \quad n << mR, \tag{15}$$

and this just corresponds to the spectrum of the potential box with the size equal to the size of the internal compact extra space. In the other case,

$$m_n = n\omega + \delta m_n, \quad \delta m_n = \frac{m\alpha^2}{2n}, \quad \omega \equiv \frac{1}{R}, \quad \alpha^2 \equiv mR, \quad \alpha^2 << n, \tag{16}$$

and here we come to the (quasi)oscillator (quasi, because n takes place of $n+1/2$ for the one-dimensional case) and (quasi)Coulomb (with $1/n$ instead of $1/n^2$ and $\alpha^2 = mR$ instead of $\alpha_c^2 = (1/137)^2$) spectra. Clearly, we can neglect the (quasi)Coulomb contribution in the region $n >> \alpha^2 \equiv mR$.

It is remarkable that KK modes of relativistic origin due to quantization of finite motion in the space of extra dimensions interpolate the non-relativistic spectrum of the potential box and the oscillator spectrum.

The spectrum of the two(a and b)-particle compound system is defined in the fundamental (input) theory by the formula

$$M_{ab}^n = m_a + m_b + \delta m_{ab}^n(m_a, m_b, G_{4+d}). \tag{17}$$

The goal of the fundamental theory is to calculate $\delta m_{ab}^n(m_a, m_b, G_{4+d})$. There is no solution of that problem in the strong interaction theory because this is a significantly non-perturbative problem. However, in the framework of Kaluza-Klein approach we can rewrite the above formula in an equivalent form

$$M_{ab}^n = m_{a,n} + m_{b,n} + \delta m_{ab,n}(m_{a,n}, m_{b,n}, g), \tag{18}$$

where $m_{a,n}, m_{b,n}$ are KK modes of particles a and b, and the quantity $\delta m_{ab,n}(m_{a,n}, m_{b,n}, g)$ can be calculated using the four-dimensional perturbation theory. Moreover, because $\delta m_{ab,n}(m_{a,n}, m_{b,n}, g) << m_{a(b),n}$, we can put with a high accuracy

$$M_{ab}^n \cong m_{a,n} + m_{b,n}, \tag{19}$$

and this allows to formulate the global solution of the spectral problem in hadron spectroscopy.

3 On the global solution of the spectral problem

According to Kaluza and Klein, we suggest that the input (fundamental) space-time $\mathcal{M}_{(4+d)}$ is represented as

$$\mathcal{M}_{(4+d)} = M_4 \times \mathcal{K}_d.$$

Let λ_n be characteristic numbers of the Laplace operator on $\mathcal{K}_d$ with a characteristic size $R_{\mathcal{K}}$

$$\Delta_{\mathcal{K}_d} Y_n(y) = -\frac{\lambda_n}{R_{\mathcal{K}}^2} Y_n(y).$$

Let $\lambda_{\mathcal{K}}$ be the set of all characteristic numbers of the Laplace operator

$$\lambda_{\mathcal{K}} \equiv \Big\{ \lambda_n : n \in \mathbb{Z}^d \equiv \underbrace{\mathbb{Z} \times \mathbb{Z} \times \cdots \times \mathbb{Z}}_{d} \Big\}. \tag{20}$$

There is one-to-one correspondence

$$\mathcal{K} \quad \Longleftrightarrow \quad (R_{\mathcal{K}}, \lambda_{\mathcal{K}}).$$

Let us consider a compound hadron system h which may decay into some channel

$$h \to a + b + \cdots + c. \tag{21}$$

We introduce the spectral mass function of the given channel by the formula

$$M_h^{ab\ldots c}(R_{\mathcal{K}},\lambda_{n_a},\lambda_{n_b},\cdots,\lambda_{n_c}) = \sqrt{m_a^2+\frac{\lambda_{n_a}}{R_{\mathcal{K}}^2}}+\sqrt{m_b^2+\frac{\lambda_{n_b}}{R_{\mathcal{K}}^2}}+\cdots+\sqrt{m_c^2+\frac{\lambda_{n_c}}{R_{\mathcal{K}}^2}}. \tag{22}$$

Now we build the Kaluza-Klein tower:

$$t_h^{ab\ldots c}(\mathcal{K}) \equiv t_h^{ab\ldots c}(R_{\mathcal{K}},\lambda_{\mathcal{K}}) \stackrel{def}{=} \Big\{ M_h^{ab\ldots c}(R_{\mathcal{K}},\lambda_{n_a},\lambda_{n_b},\cdots,\lambda_{n_c}) : \lambda_{n_i}\in\lambda_{\mathcal{K}}\,(i=a,b,...,c)\Big\}. \tag{23}$$

After that we build the Kaluza-Klein town as a union of the Kaluza-Klein towers corresponding to all possible decay channels of the hadron system h

$$\mathcal{T}_h(\mathcal{K}) \equiv \mathcal{T}_h(R_{\mathcal{K}},\lambda_{\mathcal{K}}) \stackrel{def}{=} \bigcup_{\{ab\ldots c\}} t_h^{ab\ldots c}(R_{\mathcal{K}},\lambda_{\mathcal{K}}). \tag{24}$$

We state:

$$\boxed{M_h \in \mathcal{T}_h(\mathcal{K})}. \tag{25}$$

Let $\mathcal{H}$ be the set of all possible physical hadron states. We build the hadron Kaluza-Klein country $\mathbb{C}_{\mathcal{H}}(\mathcal{K})$ by the formula

$$\mathbb{C}_{\mathcal{H}}(\mathcal{K}) \stackrel{def}{=} \bigcup_{h\in\mathcal{H}} \mathcal{T}_h(\mathcal{K}). \tag{26}$$

The whole spectrum of all possible physical hadron states we denote as $M_{\mathcal{H}}$

$$M_{\mathcal{H}} \stackrel{def}{=} \Big\{ M_h : h\in\mathcal{H}\Big\}. \tag{27}$$

We state:

$$\boxed{M_{\mathcal{H}} \in \mathbb{C}_{\mathcal{H}}(\mathcal{K})}. \tag{28}$$

Formulae (25) and (28) provide the global solution of the spectral problem in hadron spectroscopy.

Here is just the place to make some clarifying remarks. First of all, in the construction of the global solution among all possible decay channels of the hadron system h only those channels should be taken into account which contain fundamental particles and their different multi-particle compound systems in the final states, as it really should be. Appearance of non-zero KK modes of the fundamental particles and their compound systems in the final states of the decay channels is forbidden by the construction. For example, the decay channel

$$h \nrightarrow a^* + b + \cdots + c, \tag{29}$$

where a^* is a non-zero KK mode of the fundamental particle a, cannot be used in the construction. The decay channel

$$h \rightarrow A + b + \cdots + c, \tag{30}$$

where A is some multi-particle compound system which may decay into some channel with the fundamental particles $a_i (i = 1, 2, ...k)$ in the final state

$$A \rightarrow a_1 + a_2 + \cdots + a_k, \tag{31}$$

is admissible by the construction. But the decay channel

$$h \nrightarrow A^* + b + \cdots + c, \tag{32}$$

where A^* denotes some non-zero KK mode of A, is forbidden. In other words, the underlying physical principle in the construction of the global solution was the principle of non-observability of non-zero KK modes of the fundamental particles and their compound systems. According to that principle, non-zero KK modes of the fundamental particles may manifest themselves only virtually during an interaction, for example while they are staying in a compound system. Non-zero KK modes of the fundamental particles living in a compound system define the main properties of a compound system, such as mass and life time of a system. As was mentioned above, an interaction of KK modes is weak, therefore we can calculate with a high accuracy the mass of a compound system as a simple sum of the masses of KK modes. Moreover, weakly interacting KK modes result in very narrow widths of the compound states, and this phenomenon is observed in recent experiments.

The dynamics of the decays of compound systems is physically transparent: non-zero KK modes of the constituents make a transition to zero KK modes, and we observe zero KK modes as decay products. In the framework of such decay dynamics, we can estimate the widths of the compound states

$$\Gamma_n \sim \frac{\alpha_{eff}}{2} \cdot \frac{n}{R} \cdot O(1) \sim 0.4 \cdot n \,\mathrm{MeV}, \tag{33}$$

where n is the KK excitation number. The broad peaks in the hadron spectra are interpreted as an envelope of the narrow peaks predicted by the Kaluza-Klein scenario.

We have shown in the previous section that non-zero KK modes look like the states of a particle in confining potentials. Such a particle might be considered as a quasi-particle which cannot be observed without destroying a confining potential. The quasi-particle becomes a real particle by the transition of a non-zero KK mode to a zero KK mode which is equivalent to destroying the confining potential, and we observe a zero KK mode i.e. a real fundamental particle as a decay product. This consideration justifies the underlying physical principle in the construction of the global solution. In fact, here quite a new approach to the Kaluza-Klein picture as a whole is presented.

4 Comparison with the experimental data

In Refs. [2, 3, 4, 6, 8, 12] we verified the global solution with the set of experimental data for the two-nucleon system, two-pion system, three-pion system, strange mesons, charmed and charmed-strange mesons, and found out that the solution described accurately the experimentally observed hadron spectra. Here we extract the main Tables from the previous papers with an addition of some new ones.

Table 1 extracted from Ref. [3] contains the theoretically calculated Kaluza-Klein tower of KK excitations for the two-nucleon system with the account that Kaluza-Klein scenario predicts $M_n^{pp} = M_n^{p\bar{p}}$, and experimentally observed mass spectra of proton-proton and proton-antiproton systems above the elastic threshold. The sources of experimental data see in literature of Ref. [3]. As is seen from Table 1, the nucleon-nucleon dynamics at low energies provides quite a remarkable confirmation of Kaluza-Klein picture. Moreover, Kaluza-Klein scenario predicts a special sort of (super)symmetry between fermionic

(dibaryon) and bosonic states, which is quite nontrivial, and Table 1 contains an experimental confirmation of this fact as well. We do hope that blanks in Table 1 will be filled in the future experimental studies.

The Kaluza-Klein tower of KK excitations for the two-pion system, extracted from Ref. [4], is shown in Table 2 where the comparison with experimentally observed mass spectrum of two-pion system is also presented; see details in [4]. Here we have only one empty cell $M_{13}^{\pi\pi}(1112 - 1114)$, and this is the subject for further careful analysis of the two-pion system. Besides, we would like to draw your attention to reference [5][2], which contains a review of the known earlier results concerning the ABC-particle observed for the first time in 1961 at Berkeley in the reaction $pd \rightarrow {}^3HeX^0$. A more precise experimental study in Ref. [5] compared to the experiments performed before allowed to establish four states in the two-pion system with the masses $M \approx 310$ MeV, $M \approx 350$ MeV, $M \approx 430$ MeV, $M \approx 550$ MeV. From expressive, historical report [6] presented at this Conference I have learned about old work [7] where the two-pion effective-mass distributions in the reaction of $\bar{p}p$ annihilation have been studied in 1962 at Berkeley too. It should be emphasized that this important paper was not cited in Ref. [5]. However, I found this article quite an interesting. First of all, as it follows from Ref. [7], measuring K_1^0 decays produced by $\bar{p} + p \rightarrow K_1^0 + K + n\pi$, a peak in the two-pion effective-mass distribution with $\Gamma/2 = 13$ MeV centered at 499 ± 3 MeV has been observed. This just corresponds to the $M_5^{\pi\pi}$-storey in Table 2. Secondly, a careful inspection of the 750-MeV region in the two-pion effective-mass distributions revealed an evidence for a double peak in this region. It was concluded that the data satisfy very well the hypothesis of two peaks at 720 and 780 MeV with $\Gamma_1 = 30$ MeV and $\Gamma_2 = 60$ MeV: the data are best fit by these two peaks. We just predict these states; see $M_8^{\pi\pi}$-storey and $M_9^{\pi\pi}$-storey in Table 2. Finally, a remarkable discontinuity in the two-pion effective-mass spectrum at 320 MeV has been observed [7] which referred to ABC peak. $M_2^{\pi\pi}$-storey in Table 2 just corresponds to this peak. Thus, Table 2 shows that there is quite a remarkable correspondence of the calculated KK excitations for the two-pion system with the experimentally observed picture in the two-pion effective-mass spectrum, and this can be considered as strong evidence of Kaluza-Klein picture of the world. It should additionally be pointed out that the $f_2(0^+2^{++})$-mesons ($M_{f_2} = 1272 \pm 8$ MeV and $M_{f_2} = 2175 \pm 20$ MeV) investigated by IHEP group accurately agree with the calculated values and excellently incorporated in the scheme of systematics provided by Kaluza-Klein picture [4].

The Kaluza-Klein tower built for the three-pion system in Ref. [8] is shown in Table 3 where the comparison with experimentally observed mass spectrum of the three-pion system is also presented. Certainly, here is a much more poor experimental data set compared to the case of the two-pion system. Nevertheless, again we see from Table 3 that there is quite a remarkable correspondence of the calculated KK excitations for the three-pion system with the experimental data where such data exist, and this fact can also be considered as an additional evidence of Kaluza-Klein picture of the world. It is pleased for us to emphasize [8] that the experimental measurement of the a_2 meson mass made by Protvino VES Collaboration with the best world precision

$$M(a_2) = 1311.3 \pm 1.6(stat) \pm 3.0(syst)\,\text{MeV}$$

[2]I thank E. Kolomeitsev for drawing my attention to this article.

is in excellent agreement with the theoretically calculated value

$$M_{10}^{\pi^+\pi^-\pi^0} = 1311.55\,\mathrm{MeV}.$$

The same is true for the ω_3 meson where the theoretically calculated mass of KK excitation in the $3\pi^0$ system $M_{13}^{3\pi^0} = 1667.68\,MeV$ is in a very good agreement with PDG AVERAGE value $M(\omega_3) = 1667\pm4\,MeV$. Moreover, it is very interesting to point out that theoretical calculation of KK excitations in the $\rho\pi$ system by the formula

$$M_n^{\rho\pi} = \sqrt{m_\rho^2 + \frac{n^2}{R^2}} + \sqrt{m_\pi^2 + \frac{n^2}{R^2}}, \quad (n = 1, 2, 3, \ldots), \tag{34}$$

where we use $m_\rho = 769.3\,MeV$ for the ρ meson mass from PDG, gives $M_{10}^{\rho\pi^0} = 1310.28\,MeV$ and $M_{10}^{\rho\pi^\pm} = 1311.67\,MeV$ which accurately agree with the experimental measurement of the a_2 meson mass provided by VES Collaboration. This means that a_2 meson may manifest itself as a configuration of the $\rho\pi$ system in the main, and this is quite nontrivial. For example, that statement is not true for the ω_3 meson. The Kaluza-Klein tower built for the $\rho\pi$-system is shown in Table 4. VES Collaboration gives $M(\pi_1(1^{-+})) = 1610 \pm 20\,MeV, (\Gamma = 290 \pm 30\,MeV$, see [9]), which is excellently incorporated in Table 4.

Table 5 corresponds to the Kaluza-Klein tower of KK excitations for the two-kaon system built in Ref. [10]. This Table apart of old experimental data contains new states recently observed in $K_S^0 K_S^0$-system and presented at this Conference in the talks [11, 12]. It is very nice to emphasize that all new states are in excellent agreement with the values predicted in the scheme of the systematics provided by Kaluza-Klein approach.

Table 6 extracted from Ref. [10] shows the Kaluza-Klein tower of KK excitations for the $K\pi$-system, and experimentally observed states are also presented here. Again we see from Table 6 that there is quite a remarkable correspondence of the calculated KK excitations for the $K\pi$-system with experimentally observed states of strange mesons.

The Kaluza-Klein towers of KK excitations for the $K\pi\pi$-system and $K\rho$-system are shown in Table 7 and Table 8 extracted from [10] where the comparison with experimentally observed mass spectrum has been presented too. As in the previous history we see from Tables 7–8 quite a remarkable correspondence of the calculated KK excitations for the $K\pi\pi$-system and $K\rho$-system with the masses of the states where such states are experimentally observed. Many blanks in Tables 7–8 indicate a wide field in experimental study of the $K2\pi$-system.

We can see new recently observed states $D_{sJ}(2317)$ decaying to $D_s\pi^0$ and $D_{sJ}(2457)$ decaying to $D_s^*\pi^0$ (discussed in comprehensive review talk presented at this Conference by Belle Collaboration [13]) in Tables 9-10 extracted from Ref. [14], where the Kaluza-Klein towers of KK excitations for the $D_s\pi$-system and $D_s^*\pi$-system are presented. We would also like to point out that $D_{sJ}(2317)$ state may occupy $M_{22}^{K^*K}$-storey in the Kaluza-Klein tower of KK excitations for the K^*K-system; see Table 11. The most impressive fact from the view point of our developed theoretical conception is the first observation of a very narrow charmonium state with a mass of $3871.8 \pm 0.7(stat) \pm 0.4(syst)\,MeV$ which decays into $\pi^+\pi^- J/\psi$ [13, 15]. It has been stressed [15] that the $\pi^+\pi^-$ invariant mass for the $M(3872)$ signal region concentrate near the ρ mass. Such state really exists, and it lives just on the second storey in the Kaluza-Klein tower of KK excitations for the

$\rho J/\psi$-system; see Table 12. As is seen from Table 12 there is a wonderful agreement of experimentally measured mass with theoretically calculated one.

Really, a wealth of new and exciting experimental data have been presented at the Conference HADRON'03. Here we would also like to concern the recent results of E835 Experiment at Fermilab presented in Aschaffenburg in talks [18] and [19]. It was pleased for us to hear that E835 have precisely measured directly the mass and width of $\eta_c(1^1S_0)$ in $\bar{p}p$ annihilation: $M(\eta_c) = 2984.1 \pm 2.1 \pm 1.0$ MeV and $\Gamma(\eta_c) = 20.4^{+7.7}_{-6.7} \pm 2.0$ MeV [18]. This new E835 measurement just filled the $M_{28}^{p\bar{p}}$-storey of the Kaluza-Klein tower shown in Table 1.

New observations of $\bar{p}p \to \chi_0 \to \pi^0\pi^0, \eta\eta$ through interference with the continuum and precise measurements of mass and width ($M(\chi_0) = 3415.5 \pm 0.4 \pm 0.07$ MeV and $\Gamma(\chi_0) = 10.1 \pm 1.0$ MeV) [18] are also nice news for us. As is seen from Table 13 χ_0-state just occupied the $M_{39}^{\eta\eta}$-storey of Kaluza-Klein tower for the $\eta\eta$ system [20].

In the same Table 13 new (preliminary though) results of E835 Collaboration [19] for the masses of resonances decaying into $\eta\eta$ have been shown by bold-face numbers. New results of E835 Collaboration [19] for the masses of resonances decaying into $\eta\pi$ have been presented in Table 14 by bold-face numbers too [20]. Asterisks in Tables 13-14 mark the states which have not been seen before. It was a great pleasure to establish that new E835 Collaboration results provided an additional excellent confirmation of our theoretical conception.

5 Mein Ruf to search new states

As was mentioned above, we have performed an analysis of experimental data on mass spectrum of the states containing strange mesons and compared them with the calculated values provided by Kaluza-Klein scenario [10]. By this way we have found out quite an interesting correspondence shown below

7-storey:

$$(?)\sigma(650) \in M_7^{\pi\pi}(640-644) \quad \longleftrightarrow \quad K^*(892) \in M_7^{K\pi}(893-898),$$

15-storey:

$$f_2(1275) \in M_{15}^{\pi\pi}(1273-1275) \quad \longleftrightarrow \quad K_2^*(1430) \in M_{15}^{K\pi}(1431-1434),$$

17-storey:

$$f_{0,2}(1430) \in M_{17}^{\pi\pi}(1435-1438) \quad \longleftrightarrow \quad K_2^*(1580) \in M_{17}^{K\pi}(1579-1582),$$

18-storey:

$$f_0(1522) \in M_{18}^{\pi\pi}(1518-1520) \quad \longleftrightarrow \quad K^*(1680) \in M_{18}^{K\pi}(1654-1657),$$

19-storey:

$$f_{0,?}(1580) \in M_{19}^{\pi\pi}(1599-1601) \quad \longleftrightarrow \quad K_3^*(1780) \in M_{19}^{K\pi}(1730-1733),$$

22-storey:

$$\eta_{?,2}(1840) \in M_{22}^{\pi\pi}(1845-1846) \quad \longleftrightarrow \quad K_0^*(1950) \in M_{18}^{K\pi}(1960-1963),$$

23-storey:

$$f_4(1935) \in M_{23}^{\pi\pi}(1927-1928) \quad \longleftrightarrow \quad K_4^*(2045) \in M_{23}^{K\pi}(2038-2040),$$

27-storey:

$$\rho_5(2250) \in M_{27}^{\pi\pi}(2256-2257) \quad \longleftrightarrow \quad K_5^*(2380) \in M_{27}^{K\pi}(2352-2354).$$

From this correspondence it follows that $K\pi$-system looks like a system built from two-pion system by replacement of some one pion with a kaon. In fact, all experimentally observed hadron states in the $K\pi$-system have the corresponding partners in the two-pion system. However, some hadron states in the two-pion system do not have the corresponding strange partners in the $K\pi$-system experimentally observed so far. That is why the further study of the $K\pi$-system is quite a promising subject of the investigations.

Concerning the three-pion system we have found out that

$$a_2(1311) \in M_{10}^{3\pi}(1309-1313), \qquad a_2(1311) \in M_{10}^{\rho\pi}(1310-1312).$$

Moreover, the strange partner of the a_2-meson is predicted which we would like to call as a_2^s-meson

$$\boxed{a_2^s(1520) \in M_{10}^{K2\pi}(1517-1523), \qquad a_2^s(1520) \in M_{10}^{K\rho}(1519-1522)}.$$

Apart of isospin $a_2^s(1520)$-meson may have the same quantum numbers as $a_2(1311)$-meson. We call up to search the $a_2^s(1520)$-meson and other strange partners of the three-pion states experimentally observed till now [16]. In this respect it seems the factory with intensive kaon beams would be a very good device to realize such programm. However, we would like to especially emphasize that recently observed states in the $K_S^0 K_S^0$ system reported at this Conference by ZEUS Collaboration [12] seem indicate on the possibility to observe at HERA the $a_2^s(1520)$-meson in the $K_S^0\pi^+\pi^-$ system where the invariant mass of the $\pi^+\pi^-$ system concentrated near the ρ-peak.

6 One comment

In the consideration made above we have used the simplest form of torus for the internal compact extra space and considered only diagonal elements in the Kaluza-Klein towers. In fact, we have established the non-trivial physical principle according to which KK modes of decay products preferably paired up in compound system when they lived on one and the same storey in Kaluza-Klein tower. However, there are exceptional cases. For example, ρ and ω mesons appear as the non-diagonal elements of the Kaluza-Klein towers:

$$m_\rho \in M_{n,m}^{\pi^1\pi^2} = \sqrt{m_{\pi^1}^2 + \frac{n^2}{R^2}} + \sqrt{m_{\pi^2}^2 + \frac{m^2}{R^2}}, \tag{35}$$

$$M_{n,m}^{\pi^+\pi^-}(n_{\pi^+}=12, m_{\pi^-}=4)=766.97\text{MeV}, \quad M_{n,m}^{\pi^+\pi^-}(n_{\pi^+}=13, m_{\pi^-}=4)=773.85\text{MeV},$$

$$M_{n,m}^{\pi^0\pi^0}(n=13, m=4)=769.78\text{MeV}, \quad M_{n,m}^{\pi^+\pi^0}(n_{\pi^+}=13, m_{\pi^0}=4)=770.92\text{MeV},$$

and

$$m_\omega \in M_{n,m,k}^{\pi^+\pi^-\pi^0} = \sqrt{m_{\pi^+}^2+\frac{n^2}{R^2}}+\sqrt{m_{\pi^-}^2+\frac{m^2}{R^2}}+\sqrt{m_{\pi^0}^2+\frac{k^2}{R^2}}, \tag{36}$$

$$M_{n,m,k}^{\pi^+\pi^-\pi^0}(n_{\pi^+}=5, m_{\pi^-}=6, k_{\pi^0}=5)=782.80\text{MeV}.$$

In general, as it follows from the observed hadron spectrum, the non-diagonal elements of the Kaluza-Klein towers are physically suppressed. Actually, the architecture of the hadron Kaluza-Klein towns is unambiguously defined by the internal compact extra space with its geometry and shapes, and we have to learn much more about the geometry and shapes of the compact internal extra space. However, one very important point in Kaluza-Klein picture is established now in a reliable way: the size of the internal compact extra space defines the global characteristics of the hadron spectra while the masses of the constituents are the fundamental parameters of the compound systems which the elements of the global structures being. The knowledge of the true internal compact extra space is the knowledge of the Everything that is the God. Our consideration made above has shown that we found out a good approximation to the true internal extra space. In our opinion, the global goal of the Natural Philosophy and the fundamental particle&nuclear physics as its part, in the future, will be in that to perceive the true internal extra space.

7 Conclusion

No doubt, the year 2003 will enter the history of particle physics as a year of fundamental discoveries. A series of new mesons have been discovered whose properties are in a strong disagreement with the predictions of conventional QCD-inspired quark potential models.

What is a remarkable here is that all new narrow states have been observed at the masses which are surprisingly far from the predictions of conventional quark potential models. It is still more remarkable that all new observed states are very narrow, their total widths being about a few MeV. The small widths were found to be in contradiction with quark model expectations as well. Does it mean the end of the constituent quark model? In any case, this means either considerable modifications in the conventional quark models have to be introduced or that completely new approaches should be applied in hadron spectroscopy.

We claim that existence of the extra dimensions in the spirit of Kaluza and Klein together with some novel dynamical ideas may provide new conceptual issues for the global solution of the spectral problem in hadron physics.

In fact, we have shown that one simple formula with one fundamental constant described more than 140 experimentally observed hadron states. This is the most impressive fact in the theoretical conception that has been developed. No quantum numbers were ascribed to the predicted states because we have made only model independent predictions for the masses of the states based on the existence of a compact internal extra space. A special model under particular consideration could give necessary information about quantum numbers of the states and (super)fine splitting of the masses.

The performed analysis allows to conclude definitely that **the experimentally observed structures in hadron spectra reveal the existence of extra dimensions and confirm the Kaluza-Klein picture of the world**.

This is certainly a remarkable fact that a series of our publications was followed by the fundamental discoveries in hadron spectroscopy mentioned above, and here we would like to point out that strong time correlation.

It is clear that further experimental studies with a higher mass resolution are of great importance. In particular, this refers to the problem of a large resonance overlap. It will also be important to learn how one could experimentally extract fine structures in a broad peak. We believe that the idea of an ultra-high resolution hadron spectrometer [6] is a vital experimental problem which can be solved in hadron physics in the nearest future. Anyway, it is too much desirable, and I do hope our most courageous wishes will come true.

I started report with Max Planck's saying, and I would like to finish it with the words of Max Planck as well.

For it fell to this (atom) theory to discover, in the quantum action, the long-sought key to the entrance gate into the wonderland of spectroscopy, which since the discovery of spectral analysis had obstinately defied all efforts to breach it.

Max Planck, Nobel Lecture, June 2, 1920

Paraphrasing Max Planck I could say that the discovery of the fundamental scale of the internal extra space with its geometry and shapes provides the long-awaited key to the entrance gate into the wonderland of hadron spectroscopy, which since the discovery of strong forces had obstinately defied all efforts to open it.

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Table 1: Kaluza-Klein tower of KK excitations of $pp(p\bar{p})$ system and experimental data.

n	M_n^{pp} MeV	M_{exp}^{pp} MeV	$M_{exp}^{p\bar{p}}$ MeV	n	M_n^{pp} MeV	M_{exp}^{pp} MeV	$M_{exp}^{p\bar{p}}$ MeV
1	1878.38	1877.5 ± 0.5	1873 ± 2.5	15	2251.68	2240 ± 5	2250 ± 15
2	1883.87	1886 ± 1	1870 ± 10	16	2298.57	2282 ± 4	2300 ± 20
3	1892.98	1898 ± 1	1897 ± 1	17	2347.45	2350	2340 ± 40
4	1905.66	1904 ± 2	1910 ± 30	18	2398.21		2380 ± 10
5	1921.84	1916 ± 2	~ 1920	19	2450.73		2450 ± 10
		1926 ± 2		20	2504.90		~ 2500
		1937 ± 2	1939 ± 2	21	2560.61		
6	1941.44	1942 ± 2	1940 ± 1	22	2617.76		~ 2620
		$\sim$1945	1942 ± 5	23	2676.27		
7	1964.35	1965 ± 2	1968	24	2736.04	2735	2710 ± 20
		1969 ± 2	1960 ± 15	25	2796.99		
8	1990.46	1980 ± 2	1990^{+15}_{-30}	26	2859.05		2850 ± 5
		1999 ± 2		27	2922.15		
9	2019.63	2017 ± 3	2020 ± 3	28	2986.22		$\mathbf{2984 \pm 2.1 \pm 1.0}$
10	2051.75	2046 ± 3	2040 ± 40	29	3051.20		
		$\sim$2050	2060 ± 20	30	3117.04		
11	2086.68	2087 ± 3	2080 ± 10	31	3183.67		
			2090 ± 20	32	3251.06		
		$\sim$2122	2105 ± 15	33	3319.15		
12	2124.27	2121 ± 3	2110 ± 10	34	3387.90		3370 ± 10
		2129 ± 5	2140 ± 30	35	3457.28		
13	2164.39	$\sim$2150	2165 ± 45	36	3527.25		$h_c(1P)(3526)$
		2172 ± 5	2180 ± 10	37	3597.77		3600 ± 20
14	2206.91	2192 ± 3	2207 ± 13	38	3668.81		

Table 2: Kaluza-Klein tower of KK excitations for two-pion system and experimental data.

n	$M_n^{\pi^0\pi^0} MeV$	$M_n^{\pi^0\pi^\pm} MeV$	$M_n^{\pi^\pm\pi^\pm} MeV$	$M_{exp}^{\pi\pi} MeV$
1	282.41	286.80	291.21	~ 300
2	316.87	320.80	324.73	322 ± 8
3	367.18	370.58	373.98	370 – i356
4	427.78	430.71	433.64	430 – i325
5	494.92	497.45	499.99	506 ± 10
6	566.26	568.48	570.70	585 ± 20
7	640.41	642.38	644.34	650 – i370
8	716.50	718.26	720.01	732 – i123
9	793.96	795.55	797.13	780 ± 30
10	872.44	873.88	875.33	870 – i370
11	951.68	953.00	954.32	955 ± 10
12	1031.50	1032.72	1033.94	1015 ± 15
13	1111.78	1112.92	1114.05	
14	1192.43	1193.49	1194.55	1165 ± 50
15	1273.38	1274.37	1275.36	1275.4 ± 1.2
16	1354.57	1355.50	1356.43	1359 ± 40
17	1435.96	1436.84	1437.72	1434 ± 18
18	1517.53	1518.36	1519.19	1522 ± 25
19	1599.24	1600.02	1600.81	$1593 \pm 8\,^{+29}_{-47}$
20	1681.07	1681.82	1682.57	1678 ± 12
21	1763.00	1763.72	1764.43	1768 ± 21
22	1845.03	1845.71	1846.40	1854 ± 20
23	1927.14	1927.79	1928.45	1921 ± 8
24	2009.32	2009.94	2010.57	2010 ± 60
25	2091.56	2092.16	2092.76	2086 ± 15
26	2173.85	2174.43	2175.01	2175 ± 20
27	2256.19	2256.75	2257.31	~ 2250
28	2338.58	2339.12	2339.66	~ 2330
29	2421.01	2421.53	2422.05	2420 ± 30
30	2503.47	2503.97	2504.48	2510 ± 30

Table 3: Kaluza-Klein tower of KK excitations for three-pion system and experimental data.

n	$M_n^{3\pi^0} MeV$	$M_n^{\pi^\pm 2\pi^0} MeV$	$M_n^{\pi^0 2\pi^\pm} MeV$	$M_n^{3\pi^\pm} MeV$	$M_{exp}^{3\pi} MeV$
1	423.62	428.02	432.42	436.81	
2	475.30	479.23	483.17	487.10	
3	550.77	554.17	557.57	560.98	$\eta(0^{-+})[547]$
4	641.68	644.60	647.53	650.46	
5	742.38	744.91	747.44	749.98	
6	849.40	851.61	853.83	856.05	
7	960.62	962.58	964.55	966.51	$\eta'(0^{-+})[958]$
8	1074.75	1076.51	1078.26	1080.02	
9	1190.95	1192.53	1194.12	1195.70	1194 ± 14
10	1308.66	1310.10	1311.55	1312.99	1311.3 ± 1.6
11	1427.51	1428.84	1430.16	1431.49	1419 ± 31
12	1547.25	1548.47	1549.69	1550.91	
13	1667.68	1668.81	1669.94	1671.08	1667 ± 4
14	1788.65	1789.71	1790.76	1791.82	1801 ± 13
15	1910.07	1911.06	1912.05	1913.04	
16	2031.86	2032.79	2033.72	2034.65	2030 ± 50
17	2153.95	2154.83	2155.70	2156.58	2090 ± 30
18	2276.29	2277.12	2277.95	2278.78	
19	2398.85	2399.64	2400.43	2401.22	
20	2521.69	2522.35	2523.10	2523.85	
21	2644.50	2645.22	2645.93	2646.65	
22	2767.54	2768.23	2768.91	2769.59	
23	2890.71	2891.36	2892.02	2892.67	
24	3013.97	3014.60	3015.23	3015.86	
25	3137.33	3137.94	3138.54	3139.14	
26	3260.78	3261.36	3261.94	3262.52	
27	3384.29	3384.85	3385.41	3385.97	
28	3507.87	3508.41	3508.95	3509.49	
29	3631.51	3632.03	3632.55	3633.08	
30	3755.21	3755.71	3756.21	3756.72	

Table 4: Kaluza-Klein tower of KK excitations for $\rho\pi$ system and experimental data.

n	$M_n^{\rho\pi^0} MeV$	$M_n^{\rho\pi^{\pm}} MeV$	$M_{exp}^{\rho\pi} MeV$
1	911.62	916.02	
2	932.19	936.13	
3	962.89	966.29	
4	1000.88	1003.81	
5	1044.23	1046.76	
6	1091.69	1093.91	
7	1142.48	1144.45	
8	1196.07	1197.83	
9	1252.08	1253.67	
10	1310.23	1311.67	$a_2(2^{++})$
11	1370.28	1371.60	
12	1432.05	1433.27	
13	1495.37	1496.50	
14	1560.10	1561.16	
15	1626.12	1627.11	$\pi_1(1^{-+})$
16	1693.32	1694.25	
17	1761.58	1762.46	
18	1830.83	1831.66	
19	1900.98	1901.77	
20	1971.95	1972.70	
21	2043.67	2044.39	
22	2116.10	2116.78	
23	2189.16	2189.81	
24	2262.81	2263.44	
25	2337.00	2337.60	
26	2411.69	2412.27	
27	2486.85	2487.41	
28	2562.43	2562.97	
29	2638.41	2638.93	
30	2714.76	2715.27	

Table 5: Kaluza-Klein tower of KK excitations for two-kaon system and experimental data.[3]

n	$M_n^{2K^0} MeV$	$M_n^{K^0K^\pm} MeV$	$M_n^{2K^\pm} MeV$	$M_{exp}^{2K} MeV$
1	998.80	994.81	990.83	
2	1009.08	1005.14	1001.20	
3	1025.99	1022.11	1018.24	1019.417±0.014
4	1049.21	1045.42	1041.63	
5	1078.32	1074.64	1070.95	$X(1070)^{(*)}$
6	1112.87	1109.30	1105.73	
7	1152.37	1148.93	1145.48	
8	1196.33	1193.01	1189.69	
9	1244.27	1241.08	1237.89	
10	1295.76	1292.69	1289.63	$\sim 1300^{(**)}$
11	1350.38	1347.44	1344.50	
12	1407.77	1404.96	1402.14	
13	1467.62	1464.91	1462.21	
14	1529.62	1527.02	1524.43	$1537^{+9}_{-8}{}^{(**)}$
15	1593.53	1591.04	1588.55	
16	1659.13	1656.74	1654.34	1655 ± 17
17	1726.22	1723.92	1721.62	$1726 \pm 7^{(**)}$
18	1794.64	1792.43	1790.22	
19	1864.24	1862.11	1859.99	1864.1 ± 1.0
20	1934.89	1932.85	1930.80	
21	2006.49	2004.52	2002.54	$X(2000)^{(*)}$
22	2078.93	2077.03	2075.12	
23	2152.14	2150.30	2148.45	$\mathbf{2150 \pm 30}$
24	2226.02	2224.24	2222.46	$\xi(2230)$
25	2300.53	2298.81	2297.08	
26	2375.60	2373.93	2372.26	
27	2451.17	2449.56	2447.94	
28	2527.21	2525.64	2524.08	
29	2603.67	2602.15	2600.63	
30	2680.52	2679.04	2677.57	

[3]The states labelled by (*) in the K_SK_S system have been reported at this Conference in the talk of E. Fadeeva [11]. The states labelled by (**) in the same K_SK_S system have been reported at this Conference in the talk of M. Barbi (ZEUS Collaboration at HERA) [12]. Bold-face number in 23-storey has been taken from the talk of I. Vorobiev presented at EPSHEP2003, 17–23 July 2003, Aachen, Germany.

Table 6: Kaluza-Klein tower of KK excitations for $K\pi$ system and experimental data.

n	$M_n^{K^0\pi^0}$MeV	$M_n^{K^0\pi^\pm}$MeV	$M_n^{K^\pm\pi^0}$MeV	$M_n^{K^\pm\pi^\pm}$MeV	$M_{exp}^{K\pi}$ MeV
1	640.60	645.00	636.62	641.02	
2	662.97	666.91	659.03	662.97	
3	696.58	699.99	692.71	696.11	
4	738.50	741.42	734.71	737.63	
5	786.62	789.16	782.93	785.47	
6	839.57	841.79	836.00	838.22	
7	896.39	898.36	892.95	894.91	$K^*(892)$
8	956.42	958.17	953.10	954.85	
9	1019.12	1020.70	1015.93	1017.51	
10	1084.10	1085.54	1081.03	1082.48	
11	1151.03	1152.35	1148.09	1149.41	
12	1219.64	1220.86	1216.82	1218.04	
13	1289.70	1290.83	1287.00	1288.13	
14	1361.03	1362.08	1358.43	1359.48	
15	1433.45	1434.44	1430.97	1431.95	$K^*_{0,2}(1430)$
16	1506.85	1507.78	1504.46	1505.39	
17	1581.09	1581.97	1578.79	1579.67	$K_2(1580)$
18	1656.08	1656.91	1653.87	1654.70	$K^*(1680)$
19	1731.74	1732.53	1729.61	1730.40	$K^*_3(1780)$
20	1807.98	1808.73	1805.93	1806.68	
21	1884.75	1885.46	1882.77	1883.49	
22	1961.98	1962.67	1960.08	1960.76	$K^*_0(1950)$
23	2039.64	2040.29	2037.80	2038.45	$K^*_4(2045)$
24	2117.67	2118.30	2115.89	2116.52	
25	2196.04	2196.65	2194.32	2194.92	
26	2274.72	2275.30	2273.06	2273.64	
27	2353.68	2354.24	2352.07	2352.63	$K^*_5(2380)$
28	2432.90	2433.44	2431.33	2431.87	
29	2512.34	2512.86	2510.82	2511.34	
30	2592.00	2592.50	2590.52	2591.02	

Table 7: Kaluza-Klein tower of KK excitations for $K\pi\pi$ system and experimental data.

n	$M_n^{K^0 2\pi^0}$MeV	$M_n^{K^0 2\pi^\pm}$MeV	$M_n^{K^\pm 2\pi^0}$MeV	$M_n^{K^\pm 2\pi^\pm}$MeV	$M_{exp}^{K2\pi}$ MeV
1	781.81	790.61	777.83	786.62	
2	821.41	829.27	817.47	825.33	
3	880.17	886.98	876.30	883.10	
4	952.39	958.24	948.60	954.45	
5	1034.08	1039.15	1030.39	1035.46	
6	1122.70	1127.14	1119.13	1123.57	
7	1216.60	1220.53	1213.15	1217.08	
8	1314.67	1318.18	1311.35	1314.86	
9	1416.10	1419.27	1412.91	1416.08	
10	1520.32	1523.21	1517.25	1520.14	
11	1626.87	1629.51	1623.93	1626.57	1629 ± 7
12	1735.39	1737.83	1732.57	1735.01	1730 ± 20
13	1845.59	1847.86	1842.89	1845.15	~ 1840
14	1957.24	1959.36	1954.65	1956.76	
15	2070.14	2072.12	2067.66	2069.63	
16	2184.13	2186.00	2181.74	2183.60	
17	2299.07	2300.83	2296.78	2298.53	
18	2414.85	2416.51	2412.64	2414.30	
19	2531.36	2532.93	2529.23	2530.81	
20	2648.51	2650.01	2646.46	2647.96	
21	2766.25	2767.68	2764.27	2765.70	
22	2884.50	2885.86	2882.59	2883.96	
23	3003.21	3004.51	3001.36	3002.67	
24	3122.33	3123.58	3120.55	3121.80	
25	3241.82	3243.03	3240.10	3241.30	
26	3361.65	3362.81	3359.98	3361.14	
27	3481.78	3482.90	3480.16	3481.28	
28	3602.19	3603.27	3600.62	3601.70	
29	3722.85	3723.89	3721.32	3722.37	
30	3843.73	3844.74	3842.25	3843.26	

Table 8: Kaluza-Klein tower of KK excitations for $K\rho$ system and experimental data.

n	$M_n^{K^0\rho}$ MeV	$M_n^{K^\pm\rho}$ MeV	$M_{exp}^{K\rho}$ MeV
1	1269.82	1265.83	
2	1278.30	1274.36	1273 $\pm$ 7
3	1292.29	1288.42	
4	1311.59	1307.81	
5	1335.93	1332.24	
6	1365.00	1361.43	
7	1398.46	1395.01	1402 $\pm$ 7
8	1435.99	1432.67	1414 $\pm$ 15
9	1477.24	1474.05	$\sim$ 1460
10	1521.89	1518.82	
11	1569.63	1566.69	
12	1620.19	1617.37	
13	1673.29	1670.58	
14	1728.70	1726.10	1717 $\pm$ 27
15	1786.20	1783.71	1776 $\pm$ 7
16	1845.59	1843.20	
17	1906.71	1904.41	
18	1969.39	1967.18	1973$\pm$8$\pm$25
19	2033.48	2031.35	
20	2098.86	2096.81	
21	2165.42	2163.44	
22	2233.05	2231.14	
23	2301.66	2299.82	
24	2371.16	2369.38	
25	2441.49	2439.76	
26	2512.57	2510.90	
27	2584.34	2582.72	
28	2656.75	2655.18	
29	2729.75	2728.22	
30	2803.29	2801.81	

Table 9: Kaluza-Klein tower of KK excitations for $D_s^{\pm}\pi$ system and experimental data.

n	$M_n^{D_s^{\pm}\pi^0}$MeV	$M_n^{D_s^{\pm}\pi^{\pm}}$MeV	$M_{exp}^{D_s^{\pm}\pi}$ MeV
1	2110.64	2115.04	$D_s^{*\pm}(2112)$
2	2129.18	2133.11	
3	2156.52	2159.92	
4	2189.87	2192.80	
5	2227.35	2229.89	
6	2267.80	2270.02	
7	2310.50	2312.47	$D_{sJ}(2317)$
8	2355.02	2356.77	
9	2401.06	2402.65	
10	2448.44	2449.88	
11	2497.02	2498.34	
12	2546.70	2547.92	
13	2597.40	2598.53	
14	2649.07	2650.13	
15	2701.66	2702.65	
16	2755.14	2756.07	
17	2809.45	2810.33	
18	2864.58	2865.41	
19	2920.50	2921.29	
20	2977.17	2977.92	
21	3034.59	3035.30	
22	3092.72	3093.40	
23	3151.54	3152.19	
24	3211.03	3211.66	
25	3271.18	3271.78	
26	3331.95	3332.53	
27	3393.34	3393.90	
28	3455.33	3455.87	
29	3517.90	3518.42	
30	3581.02	3581.53	

Table 10: Kaluza-Klein tower of KK excitations for $D_s^{*\pm}\pi$ system and $D_{sJ}(2457)$-meson.

n	$M_n^{D_s^{*\pm}\pi^0}$MeV	$M_n^{D_s^{*\pm}\pi^\pm}$MeV	$M_{exp}^{D_s^{*\pm}\pi}$ MeV
1	2254.01	2258.41	
2	2272.46	2276.39	
3	2299.65	2303.05	
4	2332.80	2335.73	
5	2370.02	2372.55	
6	2410.14	2412.36	
7	2452.47	2454.43	$D_{sJ}^+(2457)$
8	2496.56	2498.31	
9	2542.12	2543.70	
10	2588.96	2590.41	
11	2636.96	2638.28	
12	2686.01	2687.23	
13	2736.04	2737.17	
14	2786.99	2788.05	
15	2838.82	2839.81	
16	2891.50	2892.43	
17	2944.98	2945.86	
18	2999.24	3000.07	
19	3054.26	3055.05	
20	3110.01	3110.76	
21	3166.46	3167.18	
22	3223.61	3224.30	
23	3281.43	3282.08	
24	3339.90	3340.53	
25	3399.00	3399.61	
26	3458.72	3459.30	
27	3519.04	3519.60	
28	3579.95	3580.49	
29	3641.42	3641.94	
30	3703.44	3703.94	

Table 11: Kaluza-Klein tower of KK excitations for K^*K system and $D_{sJ}(2317)$-meson.

n	$M_n^{K^{*\pm}K^0}$ MeV	$M_n^{K^{*0}K^{\pm}}$ MeV	$M_{exp}^{K^*K}$ MeV
1	1392.02	1392.48	$h_1(?1^{+-})(1386 \pm 19)$
2	1400.05	1400.53	
3	1413.30	1413.82	
4	1431.57	1432.15	$f_1(0^+1^{++})(1433 \pm 0.8)$
5	1454.63	1455.27	
6	1482.18	1482.89	$\eta(0^+0^{-+})(1475 \pm 5)$
7	1513.94	1514.71	$f_1(0^+1^{++})(1512 \pm 4)$
8	1549.58	1550.42	
9	1588.80	1589.70	
10	1631.30	1632.27	$\eta_2(0^+2^{-+})(1632 \pm 14)$
11	1676.82	1677.83	
12	1725.08	1726.14	
13	1775.85	1776.95	
14	1828.91	1830.04	
15	1884.06	1885.22	
16	1941.12	1942.29	
17	1999.92	2001.11	
18	2060.32	2061.51	
19	2122.17	2123.38	
20	2185.37	2186.57	
21	2249.79	2251.00	
22	2315.35	2316.55	$D_{sJ}(2317)$ (?)
23	2381.94	2383.14	
24	2449.49	2450.68	
25	2517.92	2519.10	
26	2587.17	2588.34	
27	2657.17	2658.33	
28	2727.87	2729.01	
29	2799.22	2800.35	
30	2871.17	2872.28	

Table 12 Kaluza-Klein tower of KK excitations for $\rho J/\psi$ system and $D_{sJ}(3872)$-meson.

n	$M_n^{\rho J/\psi}$ MeV	$M_{exp}^{\rho J/\psi}$ MeV
1	3867.57	
2	3871.74	3871.8±0.7±0.4
3	3878.67	
4	3888.30	
5	3900.58	
6	3915.41	
7	3932.73	
8	3952.42	
9	3974.39	
10	3998.54	
11	4024.75	
12	4052.92	
13	4082.95	
14	4114.74	
15	4148.19	
16	4183.22	
17	4219.74	
18	4257.68	
19	4296.95	
20	4337.48	
21	4379.23	
22	4422.11	
23	4466.09	
24	4511.11	
25	4557.11	
26	4604.07	
27	4651.93	
28	4700.65	
29	4750.21	
30	4800.57	

Table 13: Kaluza-Klein tower of KK excitations for $\eta\eta$ system and experimental data.

n	$M_n^{2\eta}$ MeV	$M_{exp}^{2\eta}$ MeV	n	$M_n^{2\eta}$ MeV	$M_{exp}^{2\eta}$ MeV
1	1097.74		33	2948.47	
2	1107.10		34	3025.66	
3	1122.54		35	3103.15	
4	1143.80		36	3180.91	
5	1170.56		37	3258.94	
6	1202.47		38	3337.19	
7	1239.11		39	3415.68	$\chi_{\mathbf{0}}(\mathbf{3415.5 \pm 0.4 \pm 0.07})$
8	1280.10	$f_2(1275)$	40	3494.36	
9	1325.01	$\mathbf{2^{++}(1330 \pm 2)}$	41	3573.25	
10	1373.47		42	3652.31	
11	1425.12		43	3731.54	
12	1479.62	$\mathbf{2^{++}(1477 \pm 5)}$	44	3810.93	
13	1536.66	$f_2'(1525)$	45	3890.47	
14	1595.99	$\pi_1(1600)$	46	3970.15	
15	1657.34		47	4049.96	
16	1720.51	$\mathbf{0^{++}(1734 \pm 4)}$	48	4129.90	
17	1785.29		49	4209.95	
18	1851.53		50	4290.11	
19	1919.07		51	4370.38	
20	1987.78	$\mathbf{4^{++}(1986 \pm 5)}$	52	4450.75	
21	2057.54	$\mathbf{2^{++}(\sim 2030)}$	53	4531.21	
22	2128.24	$\mathbf{2^{++}(2138 \pm 4)}$	54	4611.76	
23	2199.80		55	4692.39	
24	2272.14		56	4773.10	
25	2345.18	$\mathbf{4^{++}(2352 \pm 8)}^*$	57	4853.89	
26	2418.86		58	4934.75	
27	2493.13	$\mathbf{6^{++}(2484 \pm 14)}$	59	5015.68	
28	2567.93		60	5096.68	
29	2643.21		61	5177.73	
30	2718.94		62	5258.85	

Table 14: Kaluza-Klein tower of KK excitations for $\eta\pi$ system and experimental data.

n	$M_n^{\eta\pi^0}$ MeV	$M_n^{\eta\pi^\pm}$ MeV	$M_{exp}^{\eta\pi}$ MeV
1	690.08	694.47	
2	711.99	715.92	
3	744.86	748.26	
4	785.79	788.72	
5	832.74	835.28	
6	884.37	886.58	
7	939.76	941.73	
8	998.30	1000.05	
9	1059.49	1061.07	
10	1122.97	1124.40	
11	1188.40	1189.72	
12	1255.56	1256.78	
13	1324.22	1325.36	$\mathbf{2^{++}(1330 \pm 2)}$
14	1394.21	1395.27	$1^{-+}(1400 \pm 20)$
15	1465.36	1466.35	$0^{++}(1474 \pm 20)$
16	1537.54	1538.47	
17	1610.63	1611.51	
18	1684.53	1685.36	$\mathbf{0^{++}(\sim 1700)}$
19	1759.15	1759.94	$\mathbf{2^{++}(1740 \pm 7)}$
20	1834.42	1835.17	
21	1910.27	1910.98	
22	1986.64	1987.32	$\mathbf{4^{++}(1986 \pm 5)}$
23	2063.47	2064.12	
24	2140.73	2141.36	
25	2218.37	2218.97	$\mathbf{4^{++}(2226 \pm 6)^*}$
26	2296.36	2296.94	
27	2374.66	2375.22	
28	2453.25	2453.79	
29	2532.11	2532.63	
30	2611.21	2611.71	